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THE HASTUS VEHICLE AND MANPOWER SCHEDULING
SYSTEM AT THE SOCIÉTÉ DE TRANSPORT DE LA
COMMUNAUTÉ URBAINE DE MONTRÉAL*

by

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ABSTRACT

Varying service levels during the day and complex work rules make transit scheduling a difficult task. Over the last 10 years, a joint university - S.T.C.U.M. - private enterprise effort has produced HASTUS, a comprehensive set of computer tools that assist in building cost-effective schedules. The HASTUS-Macro decision-support system uses linear programming methods to estimate the costs of changes in union contracts and service levels. HASTUS-Bus employs network-flow methods to generate optimal vehicle schedules. HASTUS-Micro generates operator assignments through a combination of specially-formulated heuristics and optimal matching algorithms. Annual savings in excess of \$3,000,000 (three percent of operating costs) have been reported in manpower scheduling and \$1,000,000 in vehicle scheduling. This success at the S.T.C.U.M. has led to installations in nearly 40 major cities worldwide.

Key words: Crew scheduling, vehicle scheduling, decision support system, optimization.

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RÉSUMÉ

Un niveau de service variant au cours de la journée, additionné à des règles de conventions collectives complexes tendent à rendre des plus difficiles les problèmes d'élaboration d'horaires dans les compagnies de transports en commun. HASTUS est le résultat des efforts concertés entre la S.T.C.U.M., l'Université et l'entreprise privée. Cette coopération a permis le développement d'un ensemble de programmes efficaces pour la production d'horaires, à coûts optimum. HASTUS-Macro est un système d'aide à la décision qui utilise la programmation linéaire pour estimer les changements de coûts résultant de modifications aux conventions collectives et/ou aux niveaux de service. HASTUS-Bus utilise des algorithmes de flots pour générer un horaire de véhicule optimal. HASTUS-Micro génère un horaire de chauffeur en utilisant à la fois des techniques heuristiques et un algorithme optimal de couplage. Le système a permis des économies supérieures à 3 000 000 \$ (3% des coûts d'exploitation) au chapitre des horaires de chauffeurs et 1 000 000 \$ pour les horaires de véhicules. Le succès de cette installation à la S.T.C.U.M. a été à l'origine de près de 40 autres installations à travers le monde.

Mots-clés: horaire de personnel, horaire de véhicule, système d'aide à la décision, optimisation.

The Société de transport de la Communauté urbaine de Montréal (S.T.C.U.M.) is one of North America's 10 largest public transit companies. With a staff of 8 000 and a budget of 577.5 million dollars, it serves the second largest community in Canada with a population of nearly 2 million, spread over a territory of 523 square kilometres.

To fulfill its mandate, the S.T.C.U.M. operates a fleet of 1,700 buses and employs 3,000 drivers. The network extends over 1,000 kilometres and 150 bus lines.

Since 1966, the S.T.C.U.M. has operated one of the most admired subway systems in the world. Its 750 cars circulate in an underground network of 65 stations.

In addition, the S.T.C.U.M. offers a transit service for the handicapped, with taxis and 50 special lift equipped vehicles serving nearly 1,800 passengers a day.

The S.T.C.U.M. also operates two commuter rail lines to serve the western part of the Island of Montréal. Taken together, these four modes of transportation provided 394 million passenger trips in 1988.

Managing a transit company, especially one of the size of the S.T.C.U.M., is a multi-faceted challenge. Perhaps one of the most difficult problems is to schedule vehicles and operating personnel. In addition to the 3,000 bus drivers, the S.T.C.U.M. employs 370 subway operators and 450 ticket collectors. Efficient scheduling not only improves service and working conditions, but also has an immediate impact on operating costs.

The number of buses and drivers alone explains the complexity of this task, but many more factors must be considered. A transit system must respond to large variations in service levels during

the day. Between off-peak and peak periods, the number of vehicles in service will range from a few hundred to 1,450. In addition, demands change seasonally, what with school semesters, holidays, and vacation periods.

The schedule must take into account vehicle frequencies on each route during the day, as well as differences in average vehicle speeds during peak and off-peak periods. The practice of performing driver changeovers on the road at one of the 75 relief points distributed over the territory also adds complexity. This practice is necessary to reduce vehicle deadhead time and to meet shift length rules.

Transit scheduling is accomplished in two successive operations. Vehicle scheduling is done first to provide the number of buses and subway trains needed to satisfy the required service frequencies on each route. Drivers must then be assigned to the vehicles. This second operation is called crew scheduling.

The HASTUS system is composed of three main software modules that provide optimization and data management tools for both vehicle scheduling and crew scheduling (figure 1). HASTUS-Bus is first employed to produce vehicle schedules. HASTUS-Macro is then used as an initial step in the crew-scheduling process. The Macro module can also be used as a stand-alone cost-analysis tool in union negotiations and for planning purposes. Finally, the HASTUS-Micro module produces the detailed driver schedules, using the approximate HASTUS-Macro solution as a guide.

VEHICLE SCHEDULING

Vehicle scheduling for buses, the metro, and commuter trains has three main inputs: the transit routes to be served, the vehicle frequencies required on each route, and the travel times between the major bus stops. The service planning group determines the design of the routes and the bus frequencies; the scheduling group must then plan the vehicle schedules to meet these requirements.

The output from vehicle scheduling is a set of what are called **vehicle blocks**. There are four sets of blocks: one for weekdays, one for Saturdays, one for Sundays, and one for holidays. A vehicle block is the work done by a bus from the moment it leaves the garage in the morning until the time it returns to the garage in the evening.

Vehicles leave the garage after being inspected by the driver. They then travel empty to the closest stop on the route to be served. This is called **deadhead time**. The vehicle then proceeds in one direction, say outbound, to the end of the route. Travel times vary depending on the time of day. At the end of the route, a few minutes of layover time are normally specified to allow the bus to turn around. Layover also acts as a cushion in case of delays.

The bus then returns over the route in the other direction. The bus repeats these inbound and outbound trips a certain number of times and then returns to the garage. Several buses can serve the same route at the same time in order to fulfil the service frequency requirements. For example, a given bus route could provide service every 15 minutes in the off-peak period and every five minutes in the rush hour.

Vehicle blocks thus have the general structure shown in figure 2 and consist of a series of trips in each direction. Buses can serve more than one route when this is efficient: this practice is

called interlining. Interlining can often serve to reduce overall vehicle requirements; it is, however, more difficult to schedule and operate interlined routes, so limits have been placed on the amount of interlining permitted (in some cities, interlining is not allowed at all).

Figure 3 illustrates the variation in vehicle requirements between the peak and off-peak periods. The ratio of peak to off-peak requirements is about three to one. This large variation is one of the most important characteristics of transit scheduling, and it is one of the two major complicating factors in the design of efficient vehicle and manpower schedules. The other major source of complexity is the practice of performing driver changeovers at a multitude of different locations in the city called relief points.

When building a vehicle schedule, long vehicle blocks up to 24 hours in length are first used to provide the basic service. In other words, the same bus can be in continuous service for 24 hours in a row before returning to the garage. Short vehicle blocks of two hours or more are then added to provide extra service during the morning and afternoon rush hours (figure 4).

Given the work to be done on each route, HASTUS-Bus generates an optimal set of these vehicle blocks. Constraints such as travel time, layover time and deadhead time are automatically accounted for. HASTUS-Bus uses classical network flow methods to produce this solution. The objective function to be minimized is the total number of vehicle hours required. Further discussion on the mathematical aspects of the vehicle scheduling problem in transit can be found in [Rousseau 1988].

MANPOWER SCHEDULING

Scheduling the drivers is the next operation after vehicle scheduling. Crew scheduling starts with the blocks of bus work defined by service frequencies and output from HASTUS-Bus. The longer vehicle blocks must be cut into feasible "pieces" of work since they are too long to be handled by one driver. In fact, up to nine drivers can work on the same bus during the day. Pieces of work are then combined to form legal driver workdays of the right overall length.

Numerous rules govern crew scheduling. They are the result of operational constraints and of compromises worked out over the years with the drivers' unions.

The first rule is that all pieces of work must begin and end at a relief point. A relief point is a location where driver changeovers are allowed. In Montréal, 75 potential relief points have been selected for driver convenience and safety, and they are not easily changed. They include the garages, the metro stations, and other locations around the city where parking, a restaurant, and a restroom are available.

This rule means that blocks cannot be cut into arbitrary pieces to fit the driver's shifts. Figure 5 shows the possible relief points on a set of vehicle blocks. Typically, in Montréal, each bus goes by one of these locations every hour or so.

The key to cost-effective crew scheduling is to choose the best relief times from among those available. This block-cutting procedure determines the pieces of work that will form the driver workdays.

The second major rule in crew scheduling is that driver workdays are limited to 7.5 hours over a 12.5 hour time span. While a certain percentage of split shifts is allowed, the total time from the start of the first piece in the day to the end of the last piece in the day must not exceed the 12.5 hour maximum. This span is called spread (figure 6).

Spread is a delicate question in transit, since the time not worked between the two pieces is not paid in full. Note that it is also possible to have three-piece driver days.

Other crew scheduling rules include the provision for a guaranteed 7.5 hour workday. This means that if the pieces of work in a driver's day add up to less than 7.5 hours, the driver is paid the guaranteed hours anyway. In addition, to complicate matters, overtime over 7.5 hours may not be built into normal driver workdays.

The goal of crew scheduling can thus be succinctly summarized as the production of a minimum cost set of driver workdays given both a set of vehicle blocks to be covered and the rules defining the restrictions on driver shifts.

THE HASTUS APPROACH TO MANPOWER SCHEDULING

Before HASTUS, two basic approaches were used in automated transit crew scheduling: the set-covering method and the RUCUS method. They remain today the only major alternatives to the HASTUS decomposition approach. [Wren 1981] provides an overview of research in this area.

The set covering technique is the most natural [Heurgon 1972]. Basically, this method involves trying to fit together all possible combinations of pieces into driver workdays, and choosing the least cost solution. The problem here is that the potential number of combinations is nearly infinite.

We once calculated that the total number of variables in the S.T.C.U.M. scheduling problem was in the tens of billions. Needless to say, it is impractical to solve the complete problem directly, even on the fastest computers.

To make any but the smallest of problems manageable, the set covering approach requires front-end heuristics to reduce the search space. What is normally done is to try to limit this search space to solutions that are similar to the existing manual solution. This has the fairly obvious disadvantage of precluding new and radically different solution strategies.

The RUCUS method, for its part, basically reproduces existing manual methods. These consist of cutting vehicle blocks and systematically matching early and late pieces into driver work days according to various rules of thumb [Bennington and Rebato 1975].

HASTUS grew out of a joint S.T.C.U.M. - University of Montréal research project that had the initial goal of finding an efficient alternative to these two methods. The initial research began as part of master's and doctoral work by J.-Y. Blais starting in 1975.

On the strength of conclusive trials of the prototype system developed between 1976 and 1980, full implementation at the S.T.C.U.M. began in 1981. [Lamont 1986] describes the transition from manual to computer-assisted scheduling.

One of the major innovations in the HASTUS system is the use of a two-phase approach: an initial HASTUS-Macro solution is used to guide the HASTUS-Micro solution. HASTUS-Macro thereby reduces the solution search space, using an efficient mathematical approximation rather than heuristic methods.

The originality of HASTUS lies in thus how it decomposes the problem into manageable subproblems while retaining enough of the essential structure to produce good results. HASTUS does not claim to be optimal, but in general it saves from one to four percent of total manpower costs when compared to either manual solutions or those generated by other systems.

These savings have been confirmed by parallel tests conducted by several transit authorities in North America and Europe when evaluating HASTUS [Rousseau and Blais 1988]. These tests showed that HASTUS saves, in fact, approximately 20 percent of unproductive paid time, in other words 20 percent of what can possibly be saved over the existing solutions. Savings originate from a global reduction in travel time to and from relief points, as well as in the bonuses paid for excessive spread and for overtime required to cover left-over pieces of work. In addition, HASTUS requires two to three times less computer time than the other methods.

HASTUS-MACRO: A FIRST SOLUTION

The Macro method is based on three approximations or relaxations that permit a rapid first solution to the manpower scheduling problem. The relaxations include allowing fractional results in terms of driver requirements; approximating bus schedules by assuming that all bus blocks start on the half-hour, and ignoring travel time.

The first relaxation is fairly evident and is often used in linear programming formulations.

The second relaxation means that vehicle requirements are expressed on a half-hourly basis. Changeovers are assumed to occur at the beginning of each 30-minute period, rather than at the real times required by the availability of relief points.

The last simplification in the model effectively ignores travel time to and from the various relief points. What this relaxation assumes, therefore, is that drivers can change vehicles instantaneously. Travel to relief points is addressed later, however, during the Micro phase.

These approximations permit an idealized solution to be generated rapidly using standard LP techniques. The number of variables is reduced to the 3,000 range because of the relaxations used. Certain constraints, corresponding to what are called valid inequalities in mathematical terms, are added to ensure a usable solution. The Appendix provides details of the mathematical formulation of HASTUS.

The input to HASTUS-Macro is the set of vehicle blocks to be covered, approximated to the nearest half-hour. The rules of the union contract constitute the second major input. The output from HASTUS-Macro consists of a list of pieces of work that provide the

least expensive way of covering the idealized vehicle blocks. These pieces of driver work are described by their starting times and their length (figure 7). This list later acts as guide for HASTUS-Micro when it cuts the real vehicle blocks.

In addition, HASTUS-Macro provides an accurate lower bound to the global cost of the solution that can be achieved later by detailed scheduling. It has been shown empirically that the cost of the final HASTUS-Micro solution is normally less than one percent above that provided by HASTUS-Macro. Knowing this, transit companies frequently employ HASTUS-Macro as a stand-alone tool for what-if type analysis [Blais, Rousseau 1982].

HASTUS-MICRO: GENERATING DRIVER WORKDAYS

After using HASTUS-Macro to produce an approximate solution, the S.T.C.U.M. uses the HASTUS-Micro module to produce the detailed shifts to be assigned to the drivers. HASTUS-Micro is itself divided into three steps.

For the first step in HASTUS-Micro, we developed a technique to partition the real vehicle blocks to follow as closely as possible the idealized solution provided by HASTUS-Macro. Blocks are first cut at available relief points to produce an initial solution. This solution is improved by recutting the vehicle blocks one after the other in an iterative fashion until the solution becomes stable. During this process, the objective function to be minimized is the square of the difference between the number of pieces of each type in the Micro and Macro solutions.

HASTUS-Micro cuts each vehicle block into feasible pieces of work by selecting a path from the beginning to the end of the block over the arcs representing all the potential pieces that can be cut from the block. The path over these arcs that most improves the objective function is selected by a shortest path algorithm [Figure 8] (see appendix for more details).

Our next innovation was to develop an extremely fast matching algorithm [Lessard, Minoux, Rousseau 1989]. We needed this second step in HASTUS-Micro to hook together the pieces of work generated by the block-cutting procedure, while respecting a number of constraints, including maximum spread and travel time to and from the relief points. This new algorithm is also based on a network flow formulation and generates integer solutions. The output from this step is a set of feasible, minimum-cost workdays.

In the third and final step in the process, the schedulers can test their ideas on ways to marginally improve the solution, both from

a cost-savings point of view and in terms of improving working conditions for the drivers. Here again, HASTUS differed from other systems that produced mainly take-it-or-leave-it type solutions with no what-if? capabilities. With HASTUS, the scheduler can use selection files to guide the final computer solution. These files and their usage are discussed later.

This is a major selling point of the HASTUS product, because it gives the experienced scheduler considerable control over the final solution.

To sum up the major scheduling steps, HASTUS-Bus first produces an optimal set of vehicle blocks to cover the transit route work to be done; HASTUS-Macro then indicates the most efficient block-cutting strategy; and finally HASTUS-Micro cuts the vehicle blocks at the real relief points into pieces of work and then matches these pieces into valid driver workdays.

IMPROVEMENT IN SERVICE COST AND QUALITY

The S.T.C.U.M.'s transition to HASTUS was a dramatic step because vehicle and crew scheduling are at the very heart of day-to-day operations. Good scheduling improves service quality, reduces costs, and improves morale.

Given this, S.T.C.U.M. management phased in HASTUS over a two-year period as the various bus and metro divisions successfully switched from manual to computerized methods. The system proved reliable, both as a tool for estimating manpower costs and for producing, on-time, efficient operational schedules.

The system reduced errors saving at least \$100,000 yearly in unnecessary wages while eliminating sources of potential labour-management friction. This can occur, for example, if timetable errors cause vehicles to run late or drivers to miss their changeovers.

The scheduling department successfully adapted to the new technology. The system is easy to use and the initial training program taught all 16 schedulers how to use HASTUS effectively. The schedulers quickly recognized that HASTUS enhanced their jobs, as much of the detailed clerical work has been taken over by the computer. Overtime in the scheduling area due to last-minute rushes was reduced by two-thirds.

HASTUS has enhanced scheduler morale because scheduling has become a more interesting and challenging job. The last vacancies, for example, attracted nearly 100 internal applications.

Taking the initial solution generated by HASTUS, schedulers can work with the various parameters to reduce costs further, often saving an additional one-half percent or more in driver costs, while improving schedule quality in terms of working conditions and

ease of operation.

The scheduler can improve the HASTUS final solution by using selection files. For example, suppose a driver has to work on a Sunday, from 6 to 11:30 and then from 15:30 to 17:30. This kind of workday is legal but is not popular with the drivers and often leads to increased absenteeism, especially for the small second piece of work. The scheduler in such a case can use HASTUS selection files to express the fact that the corporation is willing to see an increase in overall cost of, for example, five dollars to get rid of this kind of workday. At this point, the system then solves the matching problem again using these penalties in addition to the costs of the workdays. This reoptimization is fast enough to be done in an interactive session.

The scheduler can often rematch pieces of work so as to eliminate many of what are considered less favorable driver days. A penalty of as little as one cent can often be enough to make a worthwhile change. This is where the scheduler's skill and experience come to play. The scheduler indicates the various "soft" rules and preferences; the computer does the calculations and indicates the cost.

The first impacts the S.T.C.U.M. clearly measured were thus in the crew scheduling area. The scheduling department carefully compared the costs of parallel manual and computer-generated solutions. This study showed that the HASTUS system paid for itself in less than three months.

HASTUS also proved effective in the vehicle scheduling area. The whole process of preparing vehicle timetables is now much faster. Interlining was increased by 10 percent over previous levels, saving more than 15 buses. HASTUS-Bus also helps to produce better quality schedules, that is, fewer tight schedules with improved inter-route synchronisation.

The last component, HASTUS-Macro, has become an indispensable tool for service planning, budgeting, negotiations with the union, and discussions with the corporation's board of directors. HASTUS-Macro has given the S.T.C.U.M. the ability to perform in a few minutes the kind of what-if analyses that used to take six weeks to do manually. This tool has without a doubt saved millions of dollars by enhancing the input to the executive decision-making process.

During the last round of negotiations, for example, the S.T.C.U.M. reached an agreement with the union to reduce the work day from eight to seven and a half hours and to reduce maximum spread from 13 to 12.5 hours. This combination was not the one favored by the management negotiating team before completing an impact study with HASTUS-Macro. In fact, HASTUS-Macro showed that the original negotiating position would have raised costs much more than had been initially estimated.

The impact of HASTUS at the S.T.C.U.M. can be summarized by the following extracts from an interview given by Louise Roy, the PDG of the S.T.C.U.M.

"Technical innovations are fundamental to mass transit systems in order to face the many challenges of improving operations performance and service to our customers. At the S.T.C.U.M., HASTUS has meant not only the integration of a complex technology into our everyday operations, it has also meant improving the working conditions of our bus drivers, our metro operators, and our ticket takers. It is not only a technical solution to a technical problem. It is a whole management approach to improving working conditions and customer service.

HASTUS has not only made possible a recurring economy of \$4,000,000 a year by reducing manpower costs and vehicle requirements, it has at the same time improved

schedulers' moral because they have a better tool to work with. It has also permitted the S.T.C.U.M. to be known around the world in cities like Singapore, Los Angeles, Boston, Stockholm, Barcelona, Helsinki, and Ottawa. It is with a deep sense of pride that we note that the expertise that we have developed is exportable."

A SUCCESS AT THE S.T.C.U.M., NOW USED IN 13 COUNTRIES

HASTUS has been a complete success at the S.T.C.U.M. The system is completely integrated into the corporation's day-to-day business at all levels, from the scheduling department to the executive board room and in each divisional garage. New features and extensions are constantly being added to keep the S.T.C.U.M. at the forefront of the current modernization effort underway in a transit industry that must increasingly compete for market share with other modes of transportation.

As stated by Madame Roy, the S.T.C.U.M. is proud to be associated with the success of HASTUS on the world market. It is perhaps noteworthy that the French character of Montréal has helped in this regard. From the start, HASTUS was designed in French, but with multilingual applications in mind. Translated first into English for the North American and U.K. markets, HASTUS also operates with Finnish, Swedish and Spanish messages and menus. HASTUS is now used in 40 cities in 13 countries world-wide. The system has successfully modeled a wide variety of transit operating practices. GIRO Inc. has also ported HASTUS to several different computer platforms including UNIX, IBM mainframes, VAX/VMS and Prime minicomputers, and micros running MS-DOS and OS/2.

APPENDIX: THE HASTUS MANPOWER SCHEDULING MODEL

This appendix describes the HASTUS manpower scheduling model and solution strategy in more detail. First, a general problem formulation is presented in order to explain the decomposition approach selected. The formulation of the HASTUS-Macro linear program is then given together with the formulation and solution strategy for block partitioning. For the description of the matching algorithm used, the readers are referred to the recent paper of [Lessard, Minoux, Rousseau 1989]. More details on the model and solution strategy can be found in [Rousseau, Lessard, Blais 1985] and [Lessard, Rousseau, Dupuis 1981].

For simplicity of presentation, we will assume here that a workday consists of no more than two pieces. In practice, we have extended this approach to take into account a certain number of workdays with three pieces of work.

We denote by a pair (i,j) a piece of work starting at time i and ending at time j . Only feasible pieces (i,j) are considered, that is pairs such that both times are included in an existing blocks. Note that (i,j) may be feasible relative to several blocks. For practical reasons, we also consider in the feasible set of pieces the null piece $(0,0)$.

A quadruplet (i,j,k,h) denotes a workday made up of the feasible pairs (i,j) and (k,h) . Only workdays (i,j,k,h) which are compatible with the union contract are considered. If either (i,j) or (k,h) is a null piece, the workday either has no break or corresponds to a left-over piece of work (often called a **tripper**).

In addition we define:

L : The number of blocks.

K : The number of distinct pieces.

- I : The set of feasible quadruplets (i,j,k,h)
- $I(i,J) \subset I$: The set of quadruplets where one of the pieces is (i,j) .
- T_ℓ : The set of all times k which are either relief times, starting times or ending times for block ℓ .
- x_{ijkh} : An integer non-negative variable corresponding to the number of drivers assigned to a workday (i,j,k,h) .
- x : The vector with components x_{ijkh} , $(i,j,k,h) \in I$.
- c_{ijkh} : The cost of the workday (i,j,k,h) ; this is normally the direct cost according to the union contract.
- y_{ij}^ℓ : A binary variable taking value 1 if piece (i,j) is used on a block ℓ as a part of a driver workday.
- y^ℓ : The vector with components y_{ij}^ℓ for all feasible (i,j) .
- y : The vector with components y^ℓ , $\ell=1,..,L$.
- Y : The set of all feasible y .

The problem can now be formulated as follows [Carraraesi, Gallo, Rousseau 1982]:

$$\text{Min } \sum_I c_{ijkh} x_{ijkh}$$

Subject to:

$$\text{i) } \sum_{I(i,j)} x_{mnkh} - \sum_{\ell} y_{ij}^{\ell} = 0 \quad \text{for all feasible } (i,j) \quad (1)$$

$$\text{ii) } Dx \geq d$$

$$\text{iii) } \sum_{i \in I_{\ell}} y_{ik}^{\ell} - \sum_{j \in J_{\ell}} y_{kj}^{\ell} = b_k^{\ell} \quad \forall k \in T_{\ell}, \ell=1, \dots, L$$

$$\text{where } b_k = \begin{cases} -1 & \text{if } k \text{ is the starting time of block} \\ +1 & \text{if } k \text{ is the ending time of block} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{iv) } x \geq 0 \text{ and integer} \\ y \in Y$$

The problem can be stated in a more compact form as:

$$\text{Min } cx$$

$$\text{i) } Bx - \sum_{\ell} y^{\ell} = 0$$

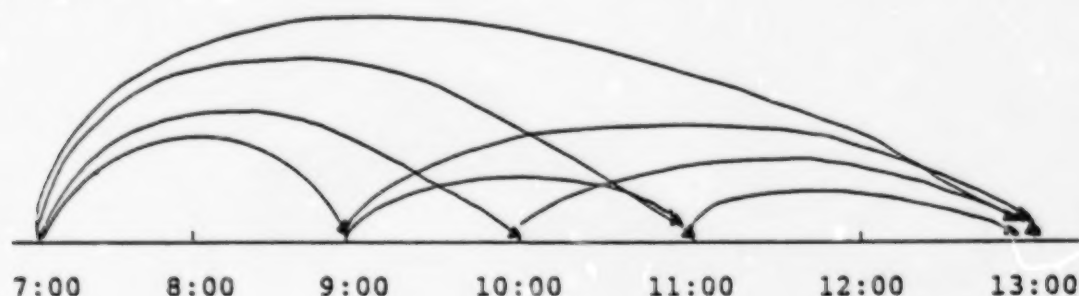
$$\text{ii) } Dx \geq d \quad (2)$$

$$\text{iii) } E y^{\ell} = b^{\ell} \quad \ell=1, \dots, L$$

$$\text{iv) } x > 0, y \in Y$$

The constraints (iii) correspond to the flow formulation used to partition each block into pieces of work. Figure A-1 illustrates the concept. The feasible pieces of work (we are assuming here a minimum length of two hours) are represented by arcs and listed in the figure. We have assumed for simplicity that a relief point exists every hour in this example. The indices k in the constraint formulation correspond to points where a change of drivers is feasible and the constraints ensure that a flow of one goes from the origin of the block (7:00) to its end (13:00), using arcs (i,j) corresponding to feasible pieces. Constraints (iii) define a flow problem on an uncapacitated network.

By constraints (i) we ensure that any feasible piece (i,j) used to partition a block will be used in a workday of type (i,j,k,h) or $(k,h,i,j,)$.



Feasible pieces are:

7 - 9	7 - 11	9 - 11	10 - 13
7 - 10	7 - 13	9 - 13	11 - 13

Figure A-1: Flow formulation of block partitioning

Constraints of type (ii) refer to any special constraints that may arise in a real life problem. Most of the time, these constraints take into account particular clauses of the union contracts such as a bound (either lower or upper) on the number of workdays without break.

Unless the problem is small, this formulation seems to be impractical. We can easily generate over 5,000 pieces and one million workday variables without considering the flow variables; moreover, there is the difficulty of solving the x_{ijkh} in integer form. In fact, this formulation is nearly equivalent in difficulty to the set covering formulation. However, starting from formulation (1), we were successful in devising an appropriate solution strategy that can handle very large problems.

THE SOLUTION STRATEGY

The chosen solution strategy is to use the obvious decomposition of the problem into the generation of a partition of each block into pieces of work (constraints (iii)) and the resulting matching problem (constraints (i)) to form workdays. Constraints (ii) also have to be satisfied. Four main steps compose this strategy.

- (1) Using a relaxation of problem (1), the HASTUS-Macro linear program is derived: this indicates the types of pieces of work that should be cut and the types of workdays that should be generated to minimize the cost of the solution [Blais and Rousseau 1982].
- (2) The blocks are partitioned into pieces so that they correspond as closely as possible to the HASTUS-Macro optimal solution (it determines the y variables). At this step a quadratic integer program is solved heuristically [Carraraesi, Gallo and Rousseau 1982].
- (3) A matching problem is set up with the pieces of work to generate workdays (the x variables). The matching problem is solved optimally using a network flow based algorithm: [Lessard, Minoux and Rousseau 1989].

- (4) A marginal improvement algorithm is used to modify the block partitions and improve the solution. Marginal costs of the pieces of work are derived from the matching problem and used to repartition the blocks [Rousseau, Lessard and Blais 1985].

A RELAXATION OF THE MODEL: THE HASTUS-MACRO APPROACH

First, the integrality of the x variables is relaxed, since several methods exist to derive reasonably good integer solutions when a continuous solution has been found. Secondly, we assume that the starting times, relief times and ending times for the blocks may only occur at predetermined times, for example every 15 or 30 minutes. This relaxation of the problem considerably reduces the number of possible pieces of work (i,j) ; however, it is important to note that a piece (i,j) may now be feasible for several blocks, which also means that x_{ijkh} may be greater than 1. All i,j,k,h are now potential relief points.

Moreover, the problem is further relaxed by requiring that the workdays selected be sufficient to cover the total requirement of drivers per time period (i.e. from one predetermined time to the next), instead of requiring that they exactly cover all the blocks individually. Using the same notation, the relaxed problem can be written as follows:

$$\text{Min } \sum_I c_{ijkh} x_{ijkh}$$

Subject to:

$$i) \quad \sum_{I(t)} x_{ijkh} \geq N_t$$

✓ $t \in T$

$$ii) \quad Dx \geq d$$

$$\text{iii) } \sum_{I(p,q)} x_{ijkh} \geq Q_{pq} \quad \forall (p,q) \text{ such that a small block exists from } p \text{ to } q \ (p,q \in T)$$

$$\text{iv) } \sum_{L(t)} x_{ijkh} \geq K_t \quad \forall t \in T$$

$$\text{v) } x_{ijkh} \geq 0 \text{ (integrality is relaxed, } i,j,k,h \in T)$$

where

T : The set of predetermined times which could be relief times, starting or ending times of blocks.

$I(t)$: The set of workdays $(i,j,k,h) \in I$ such that $i \leq t \leq j$ or $k \leq t \leq h$

$L(t)$: The set of workdays $(i,j,k,h) \in I$ with a piece starting at time t ($i = t$ or $k = t$).

N_t : The number of buses in operation during the time period starting at t .

Q_{pq} : The number of blocks from p to q .

K_t : The number of blocks starting at t .

The set of constraints (i) ensures that during all periods of the day, the number of drivers working is greater than or equal to the number of vehicles in circulation. Constraints (ii) refer to union contract constraints as previously described. Constraints (iii) require that the number of pieces of work from p to q is at least as large as Q_{pq} , the number of small blocks from p to q . A small block is defined by the user as a block that cannot be partitioned and should be allocated as one piece of work to a driver.

This generally corresponds to blocks with a duration of less than twice the minimum duration of a piece of work.

In constraint (iv), the number of pieces of work beginning at t must at least be equal to k_t , the number of blocks starting at t . Finally, x_{ijkh} is a continuous variable that can take any positive value in this relaxation. However, for x_{ijkh} to exist there must be a piece (i,j) and a piece (k,h) , each feasible with respect to at least one block.

Constraints iii) and iv) can be referred to as valid inequalities that restrict somewhat the relaxation introduced. In practice these constraints have been found to be very effective.

PARTITIONING THE BLOCKS

HASTUS-Macro is used to help generate a first feasible solution as close as possible to the lower bound indicated. This is done first by generating an initial block partition that uses similar types of pieces of work and in approximately the same number as indicated by HASTUS-Macro.

The following problem is considered:

$$\text{Min } (B\bar{x} - \sum_l (y^l))^2$$

$$\forall y^l = b^l \quad l=1, \dots, L \quad (4)$$

$$\text{where } (w)^2 = \sum_i w_i^2, \quad w \in R^n$$

and $\bar{x}$ is the optimal solution of the HASTUS-Macro problem. For simplicity of notation we consider for the moment that the pieces of work are defined on periods as in HASTUS-Macro. The problem can be easily solved using a heuristic procedure. In fact, if all but one of the decision vectors y^l are fixed ($y^l = \bar{y}^l, (l \neq r)$), the

nonlinear objective function of (4) becomes a linear function of the elements of the remaining decision vector, y^r . This function is:

$$(B\bar{x} - \sum_{i \neq r} \bar{y}^i)^2 + (e - 2(B\bar{x} - \sum_{i \neq r} \bar{y}^i))y^r$$

where e is a vector of ones and use is made of the equality $(y_{ij}^r)^2 = y_{ij}^r$. Note that the constraints of the problem in the y^r variable vector are those of the shortest path problem. We define

$$V^r(\bar{x}, \bar{y}) = (e - 2(B\bar{x} - \sum_{i \neq r} \bar{y}^i))$$

Then, in order to solve (4), a sequence of shortest path subproblems P_r is solved:

$$P_r: \text{Min } V^r(\bar{x}, \bar{y})y^r$$

Subject to:

$$E y^r = b^r$$

$$y^r \in Y$$

In summary, after finding an initial solution to (4), this solution is improved by sequentially solving P_r for $r=1, \dots, L$. The improvement phase is repeated until no improvement of the objective function has been found for a complete cycle of the P_r .

When this algorithm is applied, the objective function $(B\bar{x} - \sum \bar{y})^2$ uses pieces defined on periods, while the shortest path algorithm uses pieces defined in real time. For the evaluation of the objective function the pieces defined in real time are transformed into pieces defined on periods. The output of the algorithm is the partition of each block into pieces of work.

MATCHING THE PIECES

This step involves the generation of workdays from the pieces of work. The following matching problem is used:

$$\text{Min} \quad \sum_{I'} c_{ijkh} x_{ijkh}$$

$$\sum_{I'(i,j)} x_{mnkh} = 1, \quad \forall (i,j) \in V$$

$$x_{ijkh} = 0, 1$$

V : The set of feasible pieces resulting from the block partitioning.

I' : The set of feasible workdays using pieces from V .

$I'(i,j)$: The set of feasible workdays (m,n,h,k) I' where one of the pieces is (i,j) .

Note that, in contrast to problem (1), only the x_{ijkh} which are feasible and use pieces of work previously generated by the partitioning algorithm are generated.

The various methods used over the years to solve efficiently this matching problem has evolved into a new optimal algorithm described in [Lessard, Minoux, Rousseau [1989]]. This is based on the solution of a succession of minimum cost network flow problems.

The first (and very often the last) problem solved is the network flow problem equivalent to the linear relaxation of the matching problem. After this step, a first feasible solution for the problem is obtained.

THE MARGINAL IMPROVEMENT ALGORITHM

During this last phase, the marginal costs for each type of piece of work are calculated by reoptimizing the matching problem. Then each block is repartitioned in turn using a shortest path algorithm based on these marginal costs. The matching problem is then reoptimized using the new partition obtained: if the solution is improved, the new partition is kept. The marginal cost of each type of piece of work is recalculated regularly. Several complete iterations on all blocks are often required. For more details see [Rousseau, Lessard and Blais 1985].

CONCLUSION

Further research activities are still on-going to better solve certain types of crew scheduling problems. For example, a new approach was developed to solve problems where more than three pieces of work are necessary. In addition, a completely new algorithm, based on the set covering formulation of the problem using column generation techniques is under development at the Center for research in transportation at the University of Montréal in cooperation with GIRO. The objective of this new method is to better address smaller problems (up to 75 drivers) that are heavily constrained. This later approach is based on the work of [Desrochers, Soumis 1988].

References

Bennington, P., Rebitor, N., 1975, "Overview of the RUCUS vehicle scheduling program" in Preprints of the Workshop on Automated Techniques for Scheduling of Urban Public Transportation Companies, D. Bergmann, L.D. Bodin eds., U.S. Department of Transportation, Washington D.C.

Blais, J.-Y. and Rousseau, J.-M., 1982, "HASTUS, a model for the economic evaluation of driver's collective agreements in transit companies", INFOR, vol.20 no.1, pp. 3-15.

Blais, J.-Y., Rousseau, J.-M., 1988, "Overview of HASTUS current and future versions, in Computer-Aided Transit Scheduling, J. Daduna, A. Wren, eds., Springer-Verlag, pp. 175-188.

Carraraesi, P., Gallo, G., Rousseau, J.-M., 1982, "Relaxation Approaches to Large-Scale Bus Driver Scheduling Problems" Transportation Research B 16, pp. 383-397

Desrochers, M., Soumis, F. 1988, "Crew-opt: Crew scheduling by column generation", in Computer-Aided Transit Scheduling, Daduna, J. Wren, A. eds, Springer-Verlag, pp. 83-91.

Heurigon, E., 1972, "Un problème de recouvrement: l'habillage des horaires d'une ligne d'autobus" Revue française d'Automatique, d'Informatique et de Recherche Opérationnelle, Vol. 6, pp.13-29

Lamont, J., 1988, "The transition to computer-aided scheduling at the S.T.C.U.M." in Computer-Aided Transit Scheduling, J. Daduna, A. Wren, eds., Springer-Verlag, pp. 272-279.

Lessard, R., Minoux, M., Rousseau, J.-M., 1989, "A new algorithm for general matching problems using network-flow sub-problems" Network 19, pp. 459-479.

Lessard, R., Rousseau, J.-M., Dupuis, D, 1981, "HASTUS I: a mathematical programming approach to the bus crew scheduling problem," in Computer Scheduling of Public Transit, A. Wren, ed. North-Holland, pp. 255-267.

Rousseau, J.-M., Lessard, R., Blais, J.-Y., 1985, "Enhancements to the HASTUS crew scheduling algorithm" in Computer Scheduling of Public Transport 2, J-M. Rousseau, ed. North-Holland, pp. 295-310.

Rousseau, J.-M., 1988, "Operations research in transit: A survey", Operational Research' 87, Rand, G.K, ed., North Holland, pp. 663-685.

Wren, A., 1981, "General review of the use of computers in scheduling buses and their crews" in Computer Scheduling of Public Transport, A. Wren, ed., North-Holland, pp.3-17.

Figure 1: The service planning cycle is repeated five times yearly at the S.T.C.U.M. in order to account for seasonal variations and changing travel patterns. Vehicle and manpower scheduling are accomplished with the aid of the HASTUS software system.

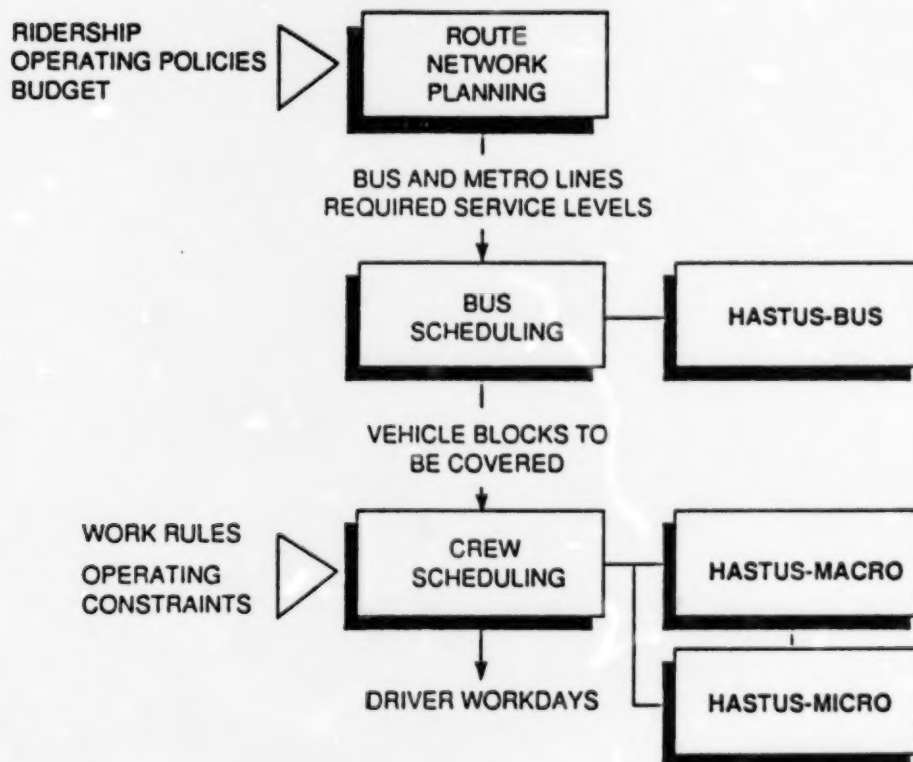


Figure 2: A vehicle block is a garage-to-garage period of work over one or more transit routes.

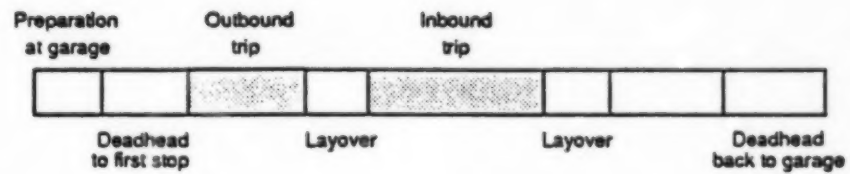


Figure 3: One inherent complexity in transit scheduling is the widely varying service demand during the day. The number of vehicles required is two to three times higher in the rush hour compared to the middle of the day.

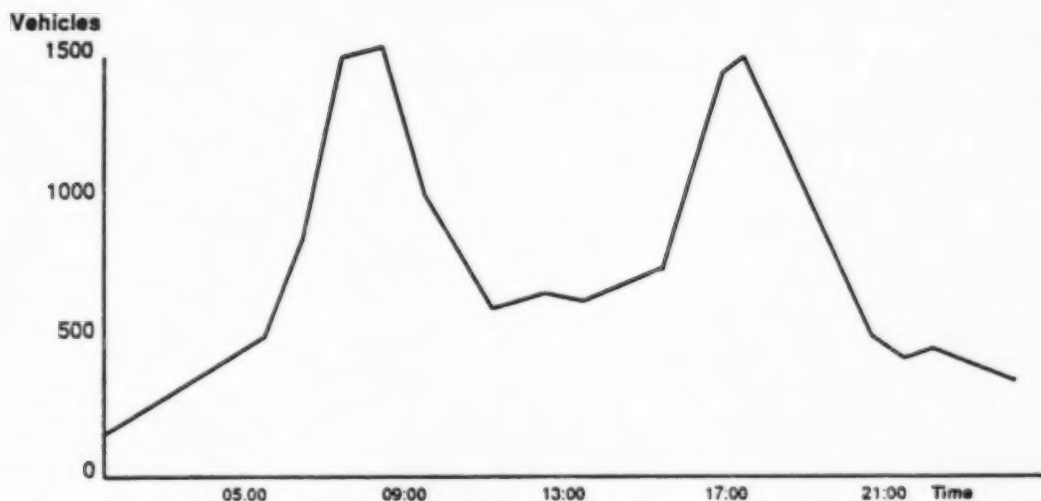


Figure 4: Four sample vehicle blocks are shown here superimposed on the vehicle requirement curve. There are typically 2,500 blocks per day.

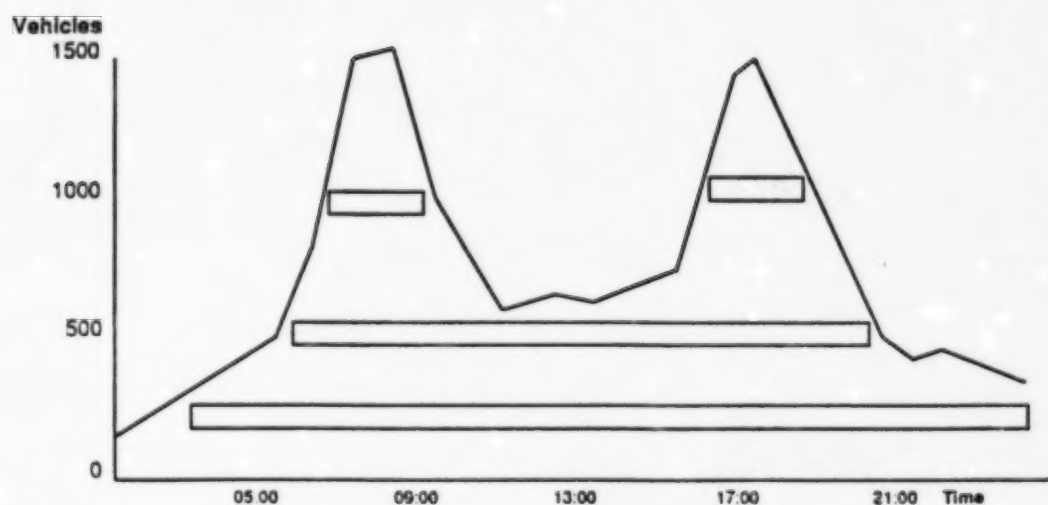


Figure 5: Relief points are locations where driver changeovers are allowed; they determine where and when vehicle blocks can be cut into pieces of work.

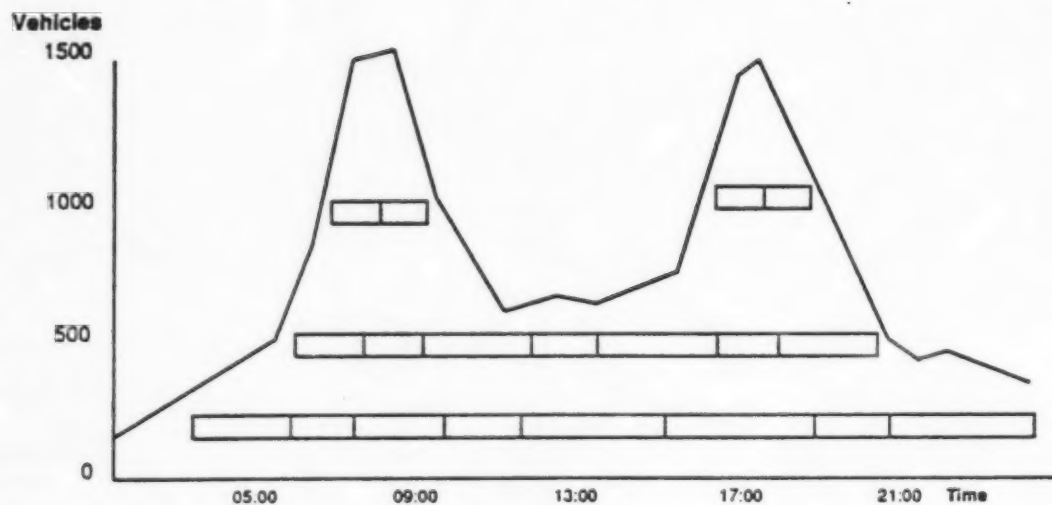


Figure 6: A driver workday is composed of one, two or three pieces of work. Here we see a split shift with two pieces, with a "spread" of approximately ten hours.

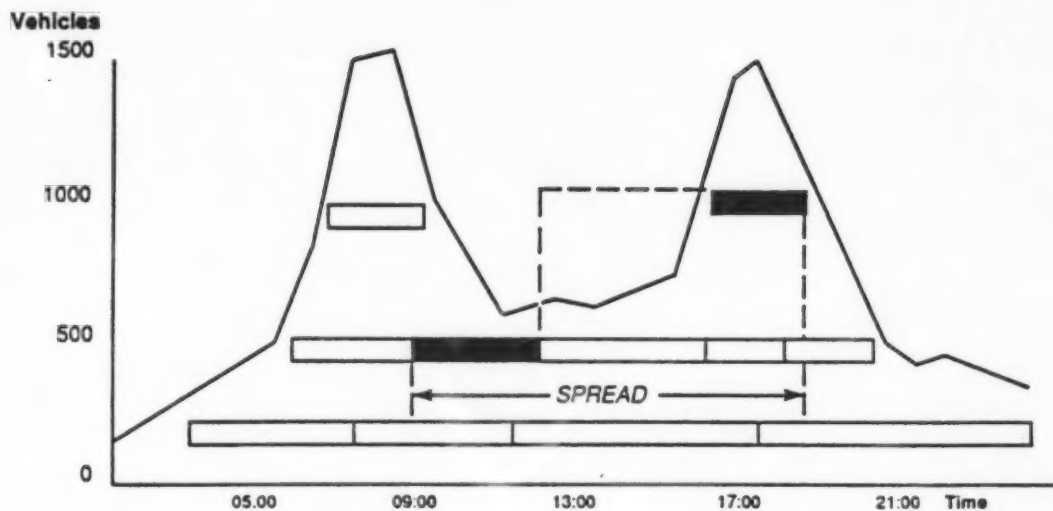


Figure 7: The HASTUS-Macro output is a set of "ideal" pieces of driver work starting at each half hour. This is used to guide the later production of optimized driver workdays.

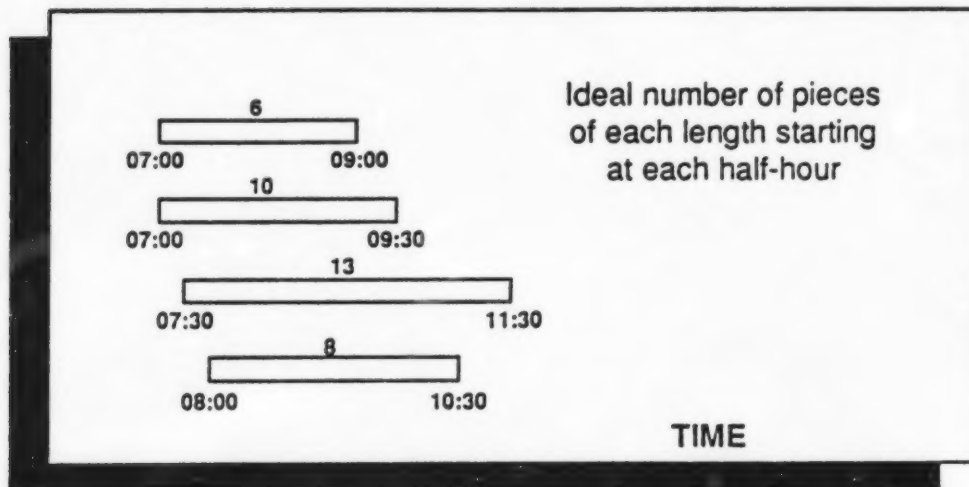


Figure 8: The HASTUS-Micro block cutting algorithm partitions vehicle blocks into pieces of work using a shortest path formulation. The arcs forming the path determine where driver changeovers will occur.

